1a) mathematical induction is a way to obtain a
18 mathematical prove for a statement S_n with n a positive interger with the following conditions: S_n with $n = 1$ (base step) is true and S_{n+1} is true whenever S_n is true (induction step) then S_n is true

b) Mathematical induction

$$S_n = \frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

"base step": $n = 1 \rightarrow \frac{1}{(2-1)(2+1)} = \frac{1}{2+1}$

$$\frac{1}{1\cdot 3} = \frac{1}{3} \qquad \text{QED}$$

"Induction step:

Suppose: $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$

$$\Rightarrow \sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1} \quad \text{(statement S1)}$$

prove: $\sum_{k=1}^{n+1} \frac{1}{(2k-1)(2k+1)} = \frac{n+1}{(2(n+1)+1)}$

$$\Rightarrow \sum_{k=1}^{n+1} \frac{1}{(2k-1)(2k+1)} = \frac{n+1}{2n+3}$$

$$\frac{1}{(2(n+1)-1)(2(n+1)+1)} + \sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n+1}{2n+3}$$

$\left(\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}\right.$ because statement

$$\frac{1}{(2n+1)(2n+3)} + \frac{n}{2n+1} = \frac{n+1}{2n+3}$$

$$\frac{1}{2n+1} + \frac{n(2n+3)}{2n+1} = n+1$$

$$\frac{1 + 2n^2 + 3n}{2n+1} = n+1$$

$$1 + 2n^2 + 3n = (2n+1)(n+1)$$

$$2n^2 + 3n + 1 = 2n^2 + 3n + 1 \qquad \text{QED}$$

We have proven by use of mathematical induction S_n is correct, because the base step and the induction holds.

2) mathematical induction

12 $S_n = 7^n - 1$ divisble by 6

"base step": $n = 1 \rightarrow 7^1 - 1$ is divisble by 6

$7 - 1$ is divisble by 6

6 is divisble by 6 $= 1$ QED

"Induction step":

Suppose : $7^n - 1$ is divisble by 6 (statement S2) is true

prove : $7^{n+1} - 1$ is divisble by 6

$7 \cdot 7^n - 1$ is divisble by 6

because $7^n - 1$ (statement S2) is divisble by 6, a multiply of this is also divisble by 6. $7 \cdot 7^n - 1 \neq 7 \cdot (7^n - 1)$

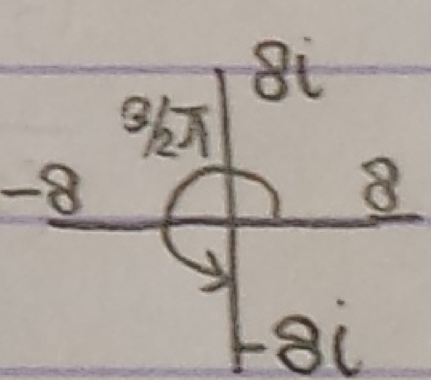
We have proven S_n by use of mathematical inductio because the base step and the induction step holds.

3) Solve : $z^3 = \frac{8}{i}$

18 $z = re^{\theta i}$

$\frac{8}{i} \rightarrow r = \sqrt{(-8)^2} = 8$

$\theta = \frac{3\pi}{2} + 2k\pi$



$\hookrightarrow 8 \cdot \frac{1}{i} = \frac{-i}{1} = -8i$

$\rightarrow 8e^{(\frac{3\pi}{2} + 2k\pi)i}$

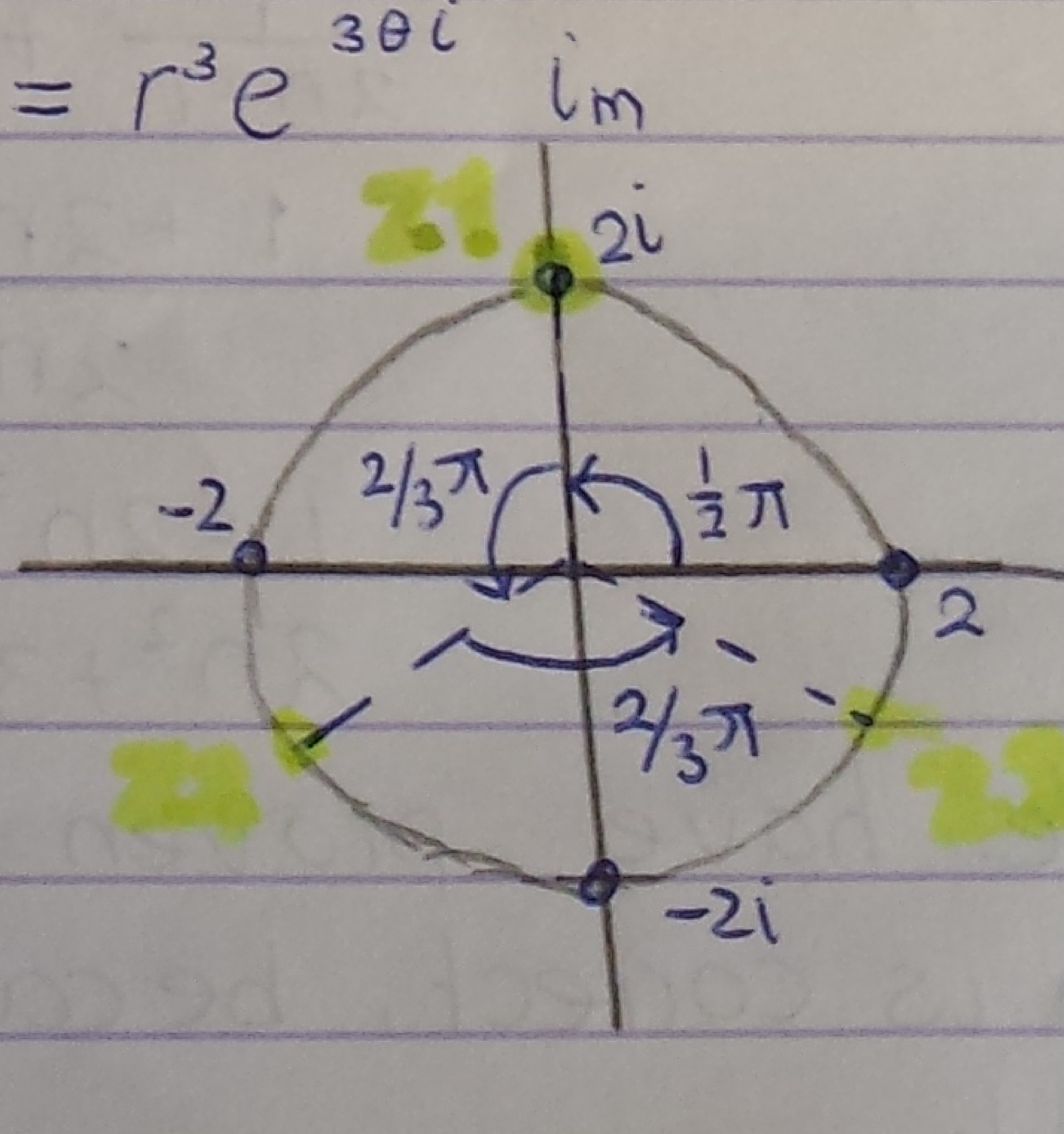
because of Elers formula $z^3 = r^3 e^{3\theta i}$

$r^3 e^{3\theta i} = 8e^{(\frac{3\pi}{2} + 2k\pi)i}$

$\rightarrow r^3 = 8 \rightarrow r = \sqrt[3]{8} = 2$

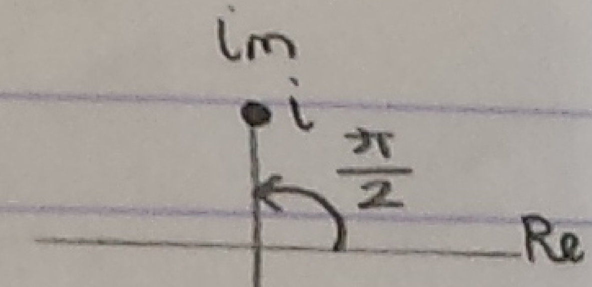
$3\theta = \frac{3\pi}{2} + 2k\pi$

$\theta = \frac{3\pi}{6} + \frac{2k\pi}{3} = \frac{\pi}{2} + \frac{2k\pi}{3}$



4) $e^{iz} = i$

16



$i = re^{i\theta}$

$r = \sqrt{1^2} = 1$

$\theta = \frac{\pi}{2} + 2k\pi$ with k a integer

$i = 1e^{i(\frac{\pi}{2}+2k\pi)}$

$e^{iz} = 1e^{i(\frac{\pi}{2}+2k\pi)}$ with k a interger

Suppose: $z = x + iy$ wit x and y real

$e^{-y+xi} = 1e^{i(\frac{\pi}{2}+2k\pi)}$ with k a interger

$e^{-y}e^{xi} = 1e^{i(\frac{\pi}{2}+2k\pi)}$ with k a interger

$\rightarrow e^{-y} = 1 \rightarrow -y = \ln(1) \Rightarrow y = -\ln(1) = 0$

$x = \frac{\pi}{2} + 2k\pi$ with k a interger

$z = x + iy$ $z = \frac{\pi}{2} + 2k\pi$ final answer

~~$\rightarrow z = e^{-0+(\frac{\pi}{2}+2k\pi)i} = e^{(\frac{\pi}{2}+2k\pi)i} \rightarrow r = 1,\ \theta = \pi/2 + 2k\pi$~~

* $\rightarrow 1(\cos(\pi/2 + 2k\pi) + i\sin(\pi/2 + 2k\pi)) = 1 \cdot (0 + i) = i$

* $z = r(\cos(\theta) + i\sin\theta)$

5) 8

$\lim_{x \to 2} x^2 = 4$

$\lim_{x \to a} f(x) = L$

$\forall \varepsilon > 0 \ \exists \delta > 0$

$0 < |x - a| < \delta, \quad |f(x) - L| < \varepsilon$

$\rightarrow a = 2$ and $f(x) = x^2$ and $L = 4$

$0 < |x - 2| < \delta, \quad |x^2 - 4| < \varepsilon$

$\underbrace{|x - 2|}_{<\delta}\ \underbrace{|x + 2|}_{<5} < \varepsilon$

$\rightarrow 5\delta < \varepsilon$

$\delta < \frac{\varepsilon}{5}$

Suppose $\delta = 1 \rightarrow |x + 2| < 5$

$\delta \leq \{\min(1, \frac{\varepsilon}{5})\}$, so the limit excists and is correct